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AN EFFICIENT SYNTHESIS-ORIENTED CAD IMPLEMENTATION OF NYQUIST STABILITY CRITERION

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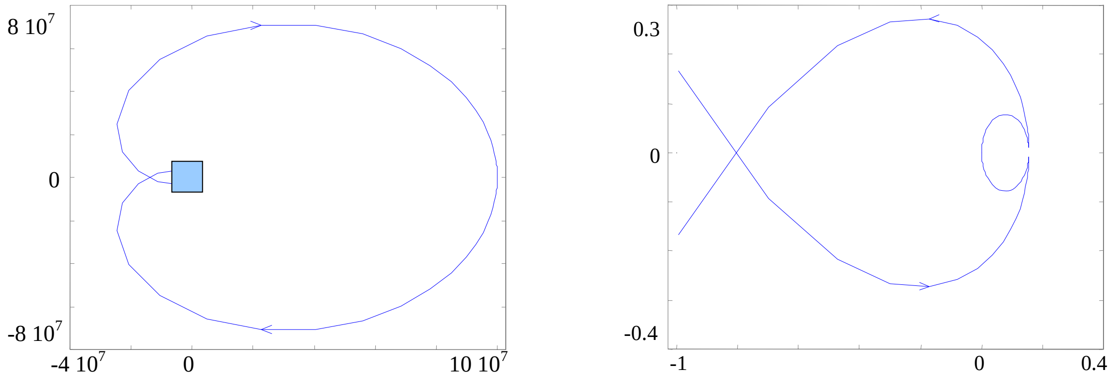
ABSTRACT

Nyquist stability criterion is largely used to determine the number of Right-Half Plane poles of feedback systems and circuits. However, visual inspection of open-loop gain polar plot is required, and automatic stability check within microwave CAD tools is not possible. In this paper, a procedure to check system and circuit linear stability by means of Nyquist criterion within CAD tools, is presented. The proposed method makes use of integral phase evaluation of a transfer function, and does not require visual inspection of Nyquist plots. The method has been successfully implemented in commercial microwave CAD tools.

1. Introduction

Control engineers largely make use of Nyquist criterion, the classical and straightforward procedure to check stability of feedback circuits and systems, to evaluate the number of closed-loop gain poles located in Right-Half Plane (RHP). Method powerfulness has suggested its use also in microwave circuit design to check stability of feedback-based topologies, and more generally of complex topologies comprising more than one active device. In fact, it has to be noted that parasitic feedback paths are present at microwave and millimetre-wave frequencies even if no explicit feedback is introduced by circuit designers. Moreover, stability check of complex topologies with several active devices and/or feedback loops, requires evaluation of the zeros of the characteristic equation of the overall circuit [1]. Run-time evaluation of design goals concerning circuit performance as well linear stability is required during optimisation-driven circuit synthesis: while the straightforward evaluation of the Rollett stability factor K is sufficient to impose unconditional stability in circuits composed of a single active device, calculation of the characteristic equation zeros, needed in the case of multi-device circuits, is a difficult task. Therefore, a rigorous criterion to impose stability of multi-device circuits with given margins, suitable to be implemented within CAD optimisation tools, is required in order to avoid cumbersome trial-and-error design procedures.

In recent years, methods based on Nyquist criterion were proposed to check Rollett proviso, requiring measured S-parameters [2], or small-signal equivalent circuits [3]–[4] of each active device. A similar methodology was also proposed to evaluate the zeros of the characteristic equation located in RHP during circuit synthesis step by means of CAD tools [5]: conditions which ensure Nyquist criterion based stability check and in the meantime do not require visual inspection, were stated: The proposed stability criterion and stability margins are suitable to be used in yield-oriented design strategies [5], which allow to avoid trial-and-error procedure in the optimisation step. These conditions are more restrictive than the Nyquist criterion, and necessary and sufficient conditions would simplify the trade-off between performance and stability. In [6] a simplified method to check Nyquist criterion was proposed, based on calculation of the winding number of the Nyquist plot with respect to a portion of the real axis. Visual inspection of the complete Nyquist plot is not required and only intersections of the Nyquist plot with the real axis are needed: unfortunately, visual inspection of the Bode plot is required to determine them. Availability of a criterion equivalent to Nyquist criterion and suitable to be checked without visual inspection of graphical plots, would permit to



a) b)

Fig. 1. a) Nyquist plot of the transfer function *G*(*j*ω) defined in Eq. 2 (unstable case), b) magnificated detail of the box in a).

improve stability testing within commercial CAD tools. In particular, such a criterion would allow to check stability in a straightforward way when complex Nyquist plots with several crossing of real axis make difficult to evaluate the number of encirclements of the critical point by visual inspection. Moreover, it could be used to analytically evaluate circuit yield with respect to stability for a feedback-based topology and/or for a multi-device circuit [2]–[5]: in this case, visual inspection is precluded by superimposition of the Nyquist plots resulting from single iterations of Monte Carlo analysis. Finally, use of this procedure would be strongly useful in conjunction with previously described design methods of multi-device circuits, which require Rollett proviso fulfilment.

In this paper, a method to evaluate the number of RHP poles of the characteristic equation of a feedback loop, without visual inspection of the Nyquist or the Bode plot, is proposed together with proper definition of stability margins, suitable to be easily implemented within CAD tools. These stability conditions can be used within a yield-oriented design procedure [5] for multi-device circuits which requires both performance optimisation and stability fulfilment. The proposed approach is based on computation of the integral phase of a given function of the open-loop gain at ω = 0 and ω → +∞, and is suitable to be implemented in microwave commercial CAD tools. In Section 2 a method to check Nyquist criterion and stability margins are presented, and in Section 3 applications to microwave circuit design by means of CAD tools are discussed. A case study is presented in Section 4 to show practical design application of the proposed method.

2. The CAD-Oriented Stability Criterion

According to Nyquist criterion, if the open-loop gain *G*(*j*ω) of a feedback system or circuit has no RHP poles, the number *N* of poles of the closed loop transfer function located in RHP equals the clockwise encirclements of the critical point (–1,0) in the Nyquist diagram. This condition is commonly checked by building the polar plot of *G*(*j*ω) and counting critical point encirclements by means of visual inspection. An equivalent way to state the previous condition, is to evaluate zeros of the characteristic equation, i.e. the numerator of *F*(*j*ω) = = 1 + *G*(*j*ω). This formulation is not usually applied to check stability, however here we'll show that it is suitable to evaluate the number *N* of RHP poles without visual inspection by means of CAD tools. In this case, encirclements number *N* can be calculated by means of the following expression:

$$N = 2\frac{\Phi_I(F(j\omega))\big|_{\omega=0} - \Phi_I(F(j\omega))\big|_{\omega=+\infty}}{2\pi}. \quad (1)$$

The function $\Phi_I(F(j\omega))$ is the integral phase of *F*(*j*ω) used to build the Bode plot of a transfer function. $\Phi_I(F(j\omega))$ is defined as the algebraic sum of phase contributions due to *F*(*j*ω) zeros and poles, and has to be evaluated at ω = 0 and ω = +∞ to calculate the number *N* of RHP poles.

As an example, we can consider a feedback system with the following expression for the loop gain:

$$G(j\omega) = K\frac{(1 - j\omega/z_1)\cdots(1 - j\omega/z_4)}{(1 - j\omega/p_1)\cdots(1 - j\omega/p_4)} \quad (2)$$

where all the poles p_i are supposed to be located in Left-Half Plane (LHP). The Nyquist plot reported in Fig. 1 for an unstable system ($K = K_1$) shows two crossing points with the negative real axis. A stable system can be found by choosing a value $K = K_2$ so that the crossing points are both at the left side or at the right side with respect to the critical point (–1,0). In Fig. 2 the Bode diagram of the function *F*(*j*ω) for the cases $K = K_1$ and $K = K_2$ is reported: in particular, $\Phi_I(F(j\omega))$ phase plots in Fig. 2 show *N* = 2 and *N* = 0 RHP poles respectively for the closed loop system, as stated in Eq. (1).

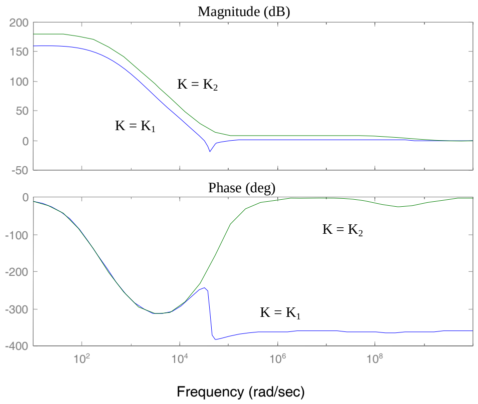


Fig. 2. Bode diagram of the transfer function 1 + $G(j\omega)$.

In conclusion, evaluation of the function $\Phi_I(F(j\omega))$ at the two points $\omega = 0$ and $\omega \to +\infty$, is sufficient to determine N without neither visual inspection of Nyquist plot nor knowledge of intersections between Nyquist plot and real axis.

Stability gain (m_g), phase (m_φ), and time (m_T) margins can be considered by applying the procedure described above to the open-loop transfer function:

$$G'(j\omega) = G(j\omega)\cdot m_g \cdot e^{-j[m_\varphi+\omega\cdot m_T]} \tag{3}$$

where $m_g > 1$, $m_\varphi > 0$, and $m_T > 0$.

If stability of the circuit with transfer function $G'(j\omega)$ is ensured, stability margins m_g, m_φ, and m_T are simultaneously guaranteed. In fact, margins m_g, m_φ, and m_T are obtained for $G(j\omega)$ if the function $G'(j\omega)$ intersects the critical point, which defines the borderline between stability and instability regions.

3. Discussion

The above described computation algorithm can be implemented in commercial CAD tools to provide stability fulfilment with given margins during optimisation-driven circuit design. In particular, Rollett proviso for stability of multi-device circuits can be checked by means of the open-loop functions defined in [2]. In such case, the stability margins m_g, m_φ, and m_T of the overall circuit are defined by considering the minima of the corresponding values obtained for the single open-loop functions. It has to be noted that implementation within commercial CAD tools of stability condition (1) requires computation of integral phase $\Phi_I(F(j\omega))$.

Integral phase function is provided within Cadence IC CAD tool [7], and can be used to check stability by means of condition (1). It has to be pointed out that availability of function $\Phi_I(\cdot)$ within CAD tools permits to check stability during circuit synthesis step, by inserting in the cost function the following single frequency (i.e. the maximal simulation frequency $f_{\max}$) stability goal:

$$\Phi_I(F(j\omega))|_{\omega=0} - \Phi_I(F(j\omega))|_{\omega=\omega_{\max}} < 2\pi - \varepsilon \tag{4}$$

where ε has to be determined according to simulation frequency step.

Some problem can arise in $\Phi_I(\cdot)$ function evaluation if this function is not directly provided within CAD tools. A method to determine the number N is to use the group delay $\tau(\omega)$ of the function $F(j\omega)$:

$$N = 2\frac{\int_0^{+\infty}\tau(\omega)d\omega}{2\pi}. \tag{5}$$

The integral and the group delay functions are provided in Cadence IC tool, but are not commonly found together in microwave CAD tools.

In such cases, integral phase function $\Phi_I(\cdot)$ can be evaluated, as in MATLAB software [8], by properly adding multiples of $\pm 2\pi$ to the four-quadrant phase function when jumps greater than $2\pi - \varepsilon$ (ε has to be chosen as stated above) are found between two adjacent frequencies.

In HP-MDS [9] microwave CAD tool, it is possible to perform only simulation post-processing, as neither the integral phase nor the group delay are available at run time: in particular, stability check within Monte Carlo analysis not requiring visual inspection of Nyquist plots is feasible.

In conclusion, stability criterion and margins proposed in this paper are suitable to be used in yield-oriented design procedures such as the one presented in [5], allowing both performance and yield optimisation, and avoiding inefficient trial-and-error design flows.

4. A Case Study

The CAD-oriented stability criterion proposed in this paper has been applied to the design of the parallel-FET amplifier shown in Fig. 3.

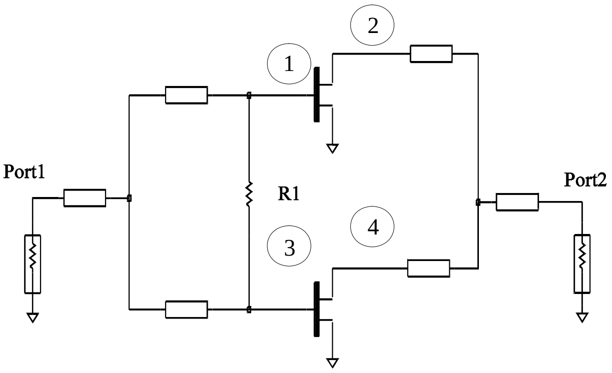
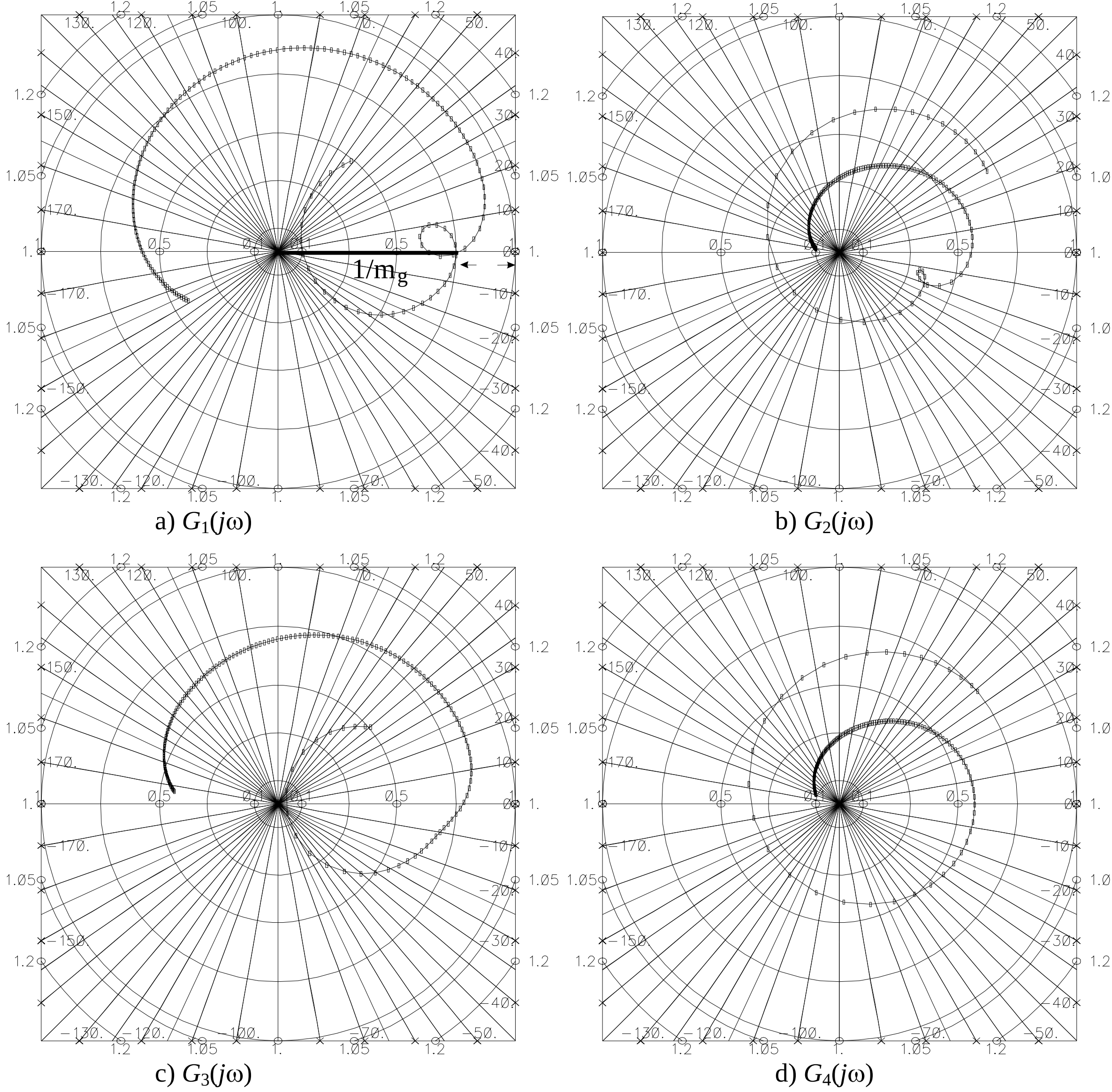


Fig. 3. Schematic circuit of the designed parallel-FET amplifier.

It is well known that this multi-device amplifier topology can show oscillations even if conditions on stability factors such as Rollett *K* are fulfilled [2]. Here, the amplifier has been designed using Toshiba JS8853-AS GaAs FETs to have a gain >8dB in the 9.2 – 10.5 GHz frequency band, return losses lower than –6 dB and a maximum gain of 11 dB at 9.8 GHz as in [2]. Cadence IC 4.4.3 CAD tool [7] was used to optimise circuit parameters according both to performance and stability design goals. As stability design goal, a gain margin $m_g >> 1.25$ was required for each of the four Nyquist plots related to the open-loop transfer functions $G_i(j\omega)$ $i = 1,2,\ldots,4$, defined in [2]. Stability condition reported in Eq. (5) where $F_i(j\omega) = 1 + G_i(j\omega)\cdot m_g$ allowed to drive CAD tool optimiser to find the optimal value for the resistor R_1, which was found to be 55 Ohm.



a) $G_1(j\omega)$

b) $G_2(j\omega)$

c) $G_3(j\omega)$

d) $G_4(j\omega)$

Fig. 4. Nyquist plots of the open-loop transfer functions $G_i(j\omega)$ of the designed parallel-FET amplifier.

After the optimisation procedure, a stability check was performed according to the procedure in [2]: Nyquist plots of the functions $G_i(j\omega)$ after optimisation are reported in Fig. 4, showing that the desired gain margin has been obtained.

5. Conclusions

An expression to check and impose linear stability of multi-device circuits by means of Nyquist criterion has been proposed. The method allows to evaluate the number of closed-loop RHP poles without visual inspection of Nyquist plot, and is useful to design stable microwave circuits based on feedback topologies and/or composed of more than one active device, avoiding trial-and-error approaches.

The procedure is suitable to be implemented in commercially available microwave CAD tools which permit post-processing of simulation data results, to check stability. Moreover, it is applicable to CAD-oriented synthesis of microwave and millimetre-wave circuits within CAD tools which make available integral phase function. The design of a 9.2 – 10.5 GHz parallel-FET amplifier has been successfully carried out and provides demonstration of the feasibility of the proposed approach.

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